Find the general solution of the differential equation $2yy'' = 1 + (y')^2$.

SCORE: ____ / 8 PTS

(2) $2yuu' = 1 + u^2$

NO INDEPENDENT VARIABLE

$u = y' \qquad uu' = y''$

$\int \frac{2u}{1+u^2}\,du = \int \frac{1}{y}\,dy$

IS $y = 0$ A SOLUTION?

$y' = 0,\ y'' = 0$

$2yy'' = 0 \neq 1 + (y')^2 = 1$

NO

$\ln|1+u^2| = \ln|y| + C$

$1 + u^2 = Cy$

$u = \pm\sqrt{Cy - 1}$

$\frac{dy}{dx} = \pm\sqrt{Cy-1}$

$\int \frac{1}{\sqrt{Cy-1}}\,dy = \pm\int dx$

IF $C = 0$, NO POSSIBLE REAL SOLUTION

$\frac{2}{C}\sqrt{Cy-1} = K \pm x$

$Cy - 1 = \left(K \pm \frac{C}{2}x\right)^2$

$y = \frac{1}{C}\left[\left(K \pm \frac{C}{2}x\right)^2 + 1\right]$

(1) EACH EXCEPT AS NOTED

Using elimination **as shown in lecture**, solve the system of differential equations **SCORE: ____ / 22 PTS**

$$5x' + 2y' + 6x - 3y = 3e^{-3t} + 6$$
$$2x' + y' + 2x - y = 4e^{-3t} - 1$$

(1) EACH

① $(5D+6)[x] + (2D-3)[y] = 3e^{-3t} + 6$

② $(2D+2)[x] + (D-1)[y] = 4e^{-3t} - 1$

APPLY $(D-1)$ TO ①

$-(2D-3)$ TO ②

$$((5D+6)(D-1) - (2D+2)(2D-3))[x] = (D-1)[3e^{-3t}+6] - (2D-3)[4e^{-3t}-1]$$

$$(D^2+3D)[x] = -9e^{-3t} - 3e^{-3t} - 6 + 24e^{-3t} + 12e^{-3t} - 3$$
$$= 24e^{-3t} - 9$$

$r = 0, -3 \rightarrow x_h = C_1 + C_2e^{-3t}$

$x_p = At + Bte^{-3t}$

$x_p' = A + B(1-3t)e^{-3t}$

$x_p'' = B(-6+9t)e^{-3t}$

$x_p'' + 3x_p' = 3A - 3Be^{-3t}$

$3A = -9 \rightarrow A = -3$

$-3B = 24 \rightarrow B = -8$

$x = C_1 + C_2e^{-3t} - 3t - 8te^{-3t}$

APPLY $-(2D+2)$ TO ①

$(5D+6)$ TO ②

$$(-(2D+2)(2D-3) + (5D+6)(D-1))[y] = -(2D+2)[3e^{-3t}+6] + (5D+6)[4e^{-3t}-1]$$

$$(D^2+3D)[y] = 18e^{-3t} - 6e^{-3t} - 12 - 60e^{-3t} + 24e^{-3t} - 6$$
$$= -24e^{-3t} - 18$$

$y = k_1 + k_2e^{-3t} - 6t + 8te^{-3t}$

$$
\begin{aligned}
2x' &= -6 + (-6c_2 - 16)e^{-3t} + 48te^{-3t} \\
+\ y' &= -6 + (-3k_2 + 8)e^{-3t} - 24te^{-3t} \\
+\ 2x &= +2c_1 + 2c_2e^{-3t} - 6t - 16te^{-3t} \\
-\ y &= -k_1 - k_2e^{-3t} + 6t - 8te^{-3t}
\end{aligned}
$$

$$
= (2c_1 - k_1 - 12) + (-4c_2 - 4k_2 - 8)e^{-3t} = 4e^{-3t} - 1
$$

$$
\begin{aligned}
2c_1 - k_1 - 12 &= -1 \\
-4c_2 - 4k_2 - 8 &= 4
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
k_1 &= 2c_1 - 11 \\
k_2 &= -c_2 - 3
\end{aligned}
$$

$$
x = c_1 + c_2e^{-3t} - 3t - 8te^{-3t}
$$

$$
y = (2c_1 - 11) - (c_2 + 3)e^{-3t} - 6t + 8te^{-3t}
$$